PAPERS


An Exact Pair for the Arithmetic Degrees whose join is not a Weak Uniform Upper Bound, in the Recursive Function Theory-Newsletters, No. 28, August-September 1982.


Axioms for Actuality, ibid., pp. 27-34.


Finite-Level Borel Games and a Problem Concerning the Jump Hierarchy, ibid, pp. 1301-1318.


SHORT ARTICLES


In the Routledge Encyclopedia of Philosophy "Recursion - Theoretic Hierarchies", "Turing Reducibility and Turing Degrees".

BOOK REVIEWS


ON THE SENSE AND REFERENCE OF A LOGICAL CONSTANT

Logicism is, roughly speaking, the doctrine that mathematics is fancy logic. So getting clear about the nature of logic is a necessary step in an assessment of logicism. Logic is the study of logical concepts, how they are expressed in languages, their semantic values, and the relationships between these things and the rest of our concepts, linguistic expressions, and their semantic values. A logical concept is what can be expressed by a logical constant in a language. So the question “What is logic?” drives us to the question “What is a logical constant?” Though what follows contains some argument, limitations of space constrain me in large part to express my Credo on this topic with the broad brush of bold assertion and some promissory gestures.

§1. Logical expressions are of three sorts: variables, logical constants, and indicators of logical force or speech-act. Let’s set aside variables, all of which are logical expressions. Let’s also set aside indicators (expressions like ‘therefore’, ‘assume that’, or ‘given a’ prefixed to a fresh free variable).

Thesis 1. Logical constants in a language constitute a natural semantic kind. Given a language L and a constant c in L’s lexicon: c is logical iff c’s sense is entirely constituted by certain of its purely syntactic roles in argumentation in L.

In particular, the distinction between logical and other constants is not merely pragmatic or conventional, to be drawn in the context of some logical or semantic inquiry merely to indicate that one will treat the expressions one calls “logical” in a distinctive way. I reject the possibility envisioned by Tarski (in 1935) that “the division of terms into logical and extra-logical” was “in greater or less degree arbitrary”.

(After giving an excellent presentation of the history of the notion of logical constanthood in [Mario Gomez-Torrente, 2002], Gomes-Torrente concludes that the project of explicating this notion, at least in terms of “unexplicated semantic or epistemic properties ... may be hopeless” (p. 32). Thesis 1 is not in

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1This is a much expanded version of the first two thirds of my talk on logicism at the Arché conference on the philosophy of mathematics, held at St. Andrews in August of 2002; that was the portion primarily on the philosophy of logic. The remaining third focused on the philosophy of mathematics.

2See [Alfred Tarski, ’35], pp. 419-20.
these terms; e.g. on my account the 1-place connective “All widows are female and” isn’t a logical constant.)

Argumentation is reasoning, or expression of reasoning, in language. Can we restrict Thesis 1 to demonstrative (i.e. deductive) argumentation? If so, c’s role in demonstrative argumentation in L determines c’s role in default, statistical and abductive argumentation, and in any other species of non-demonstrative argumentation that I’ve missed. I’m inclined to believe this. But be that as it may, in this paper I’ll restrict my attention to c’s role in demonstrative argumentation.

An expression’s sense in a language L is a concept; if not ambiguous, it’s uniquely correlated to conditions under which a fluent understander grasps the sense of that expression, as an expression of L. (Fluency here merely rules out understanding by translation into another language.) For our purposes, let’s identify grasping its sense in L with understanding its occurrences in statements in L. Of course this is rough: grasp of sense is the core of linguistic understanding, but it isn’t all of it: there is grasp of connotation, force-indication, indications of non-literal use, and perhaps more. There are degrees of grasp of a sense. I’ll say that one’s grasp is adequate if it suffices for day-to-day communicative competence. Grasp of sense is a standing mental state; when one perceives or thinks of an expression whose sense one grasps, that mental state may interact with this perception or thought to produce an occurrent mental state, which I’ll refer to as comprehension of that expression.

An argument in L is constructed from inferences, each itself an argument with no proper sub-arguments in L. An expression’s role in argumentation is codified by certain rules. I’ll understand a rule to be deductive iff it’s insensitive to context, content-neutral, and indefeasible. Context-insensitivity needs no explanation. (The “Straight Rule” for induction is context-sensitive: from \( \gamma \) m out of n random samples of \( \gamma \) s are \( \delta \) infer \( \gamma \) the probability of a \( \gamma \) being a \( \delta = m/n \) provided n is sufficiently large. Here context must determine what is sufficiently large. Of course no remotely plausible version of this rule could be content-neutral, as Nelson Goodman showed.) Content-neutrality of a rule is a matter of what counts as an instance of that rule in a given language; see §5. Indefeasibility rules out default rules. Further articulating Thesis 1, we have:

Thesis 1′. The sense of a logical constant c in L is constituted by a (note: not “the”) set \( R \) of syntactic deductive rules that govern c in L, i.e. that govern c for understanders of L.

At the risk of sounding like the poor linguist’s Chris Peacocke, let a rule R overtly primitively govern c for an L-understander S iff under normal conditions
$S$ is disposed to find inferences in $L$ that instantiate $R$ primitively compelling; this by virtue of their being instances of $R$.\(^3\) I’ll fill out this definition in the next section. Let $R$ tacitly primitively govern $c$ for $S$ iff under normal learning conditions $S$ is disposed to learn to find inferences in $L$ that instantiate $R$ overtly primitively compelling, again by virtue of their being instances of $R$, and without the distinctive cognitive process of adding a homonym to $S$’s lexicon.

Theses 2. (1) If $\mathcal{R}$ is the set of rules that constitute $c$’s sense in $L$, fully grasping $c$’s sense in $L$ is the mental state that would make its bearers be subjects for whom members of $\mathcal{R}$ overtly primitively govern $c$. (2) There is a privileged non-empty $\mathcal{R}_0 \subseteq \mathcal{R}$ whose members overtly govern $c$, making $\mathcal{R}_0$ the set of rules that overtly constitute $c$’s sense in $L$. Setting $\mathcal{R}_1 = \mathcal{R} - \mathcal{R}_0$, the members of $\mathcal{R}_1$ tacitly govern $c$ in $L$. Adequately grasping $c$’s sense in $L$ is the mental state that would make its bearers be subjects for whom members of $\mathcal{R}_0$ overtly primitively govern $c$’s sense and members of $\mathcal{R}_1$ tacitly primitively govern $c$. (3) $\mathcal{R}_0$ determines $\mathcal{R}_1$, this by a constraint that I’ll get to in §9.

The following further articulates Thesis 1.

Thesis 1”. The following are materially equivalent: (i) there is a non-empty set $\mathcal{X}$ of syntactic deductive rules that meets certain conditions (to be specified in §9) such that a full grasp of $c$’s sense in $L$ is a mental state that would make its bearers be subjects for whom members of $\mathcal{X}$ overtly primitively govern $c$; (ii) there are disjoint sets $\mathcal{X}_0$ and $\mathcal{X}_1$ of syntactic deductive rules, with $\mathcal{X}_0$ non-empty and meeting certain conditions (to be specified in §9), such that an adequate grasp of $c$’s sense in $L$ is a mental state that would make its bearers be subjects for whom members of $\mathcal{X}_0$ overtly primitively govern $c$ and members of $\mathcal{X}_1$ tacitly primitively govern $c$; (iii) $c$ is a logical constant of $L$. (Dropping the “certain conditions” opens the possibility that $c$ is what some might call a defective logical constant, and others call a meaningless expression, e.g. Prior’s ‘tonk’.)

Understand ontological relativity to be the doctrine that the range of first-order variables is relative to a framework, language, conceptual scheme, or postulational situation. Applied to variable-binding logical constants, Thesis 1 and its above elaborations are incompatible with ontological relativity, at least if the sense of such a constant uniquely determines the range of the variables it binds. I’m inclined to embrace that ‘if’-clause, and so to reject ontological relativity.

§2. To characterize deductive rules, we must consider two kinds of inferences.

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\(^3\)See [Christopher Peacocke, ’87] and [Christopher Peacocke, ’92] pp. 143-5, for the motivation for the “by virtue of” clause. Note: Peacocke discusses thought; but his motivations carry over to linguistic understanding.
A formula-inference in $L$ goes from a set $\Delta$ of formulas to a set $\Gamma$ of formula, all in $L$. Represent such an inference as $\Delta \Rightarrow \Gamma$. If $\Gamma = \{\varphi\}$, I’ll omit the curly-brackets, as is customary. (‘$\Rightarrow$’ is a function-constant added to English to form terms that designate inferences when completed by appropriate terms on the left and right. Neither $\Delta \Rightarrow \Gamma$ nor $\Delta \Rightarrow \varphi$ is a linguistic expression; so corner-quotes around ‘$\Delta \Rightarrow \Gamma$’ or ‘$\Delta \Rightarrow \varphi$’ would be incorrect. We could define inferences to be ordered pairs, letting $\Delta \Rightarrow \Gamma = (\Delta, \Gamma)$ and $\Delta \Rightarrow \varphi = (\Delta, \varphi)$.) As Gentzen was the first to appreciate, the phenomenon of discharging assumptions in ordinary reasoning makes it useful to consider inferences from formula-inferences to formula-inferences; call them sequent inferences.

My construal of finding an inference compelling is sentimental. For $S$ to find a single-conclusion formula-inference $\Delta \Rightarrow \varphi$ overtly compelling is (1) for $S$ to be disposed to feel compelled to accept $\varphi$ given that $S$ accepts $\Delta$ and comprehends $\varphi$, and (2) if $\varphi \notin \Delta$, for that feeling to be brought about by a process (2.1) initiated by $S$’s acceptance of $\Delta$ and $S$’s comprehension of $\varphi$, and (2.2) not depending on $S$’s prior acceptance of $\varphi$. Here, to accept a set of formulas $\Delta$ is to accept each member of $\Delta$, all at the same time.

For a definition of finding $\Delta \Rightarrow \varphi$ overtly primitively compelling, add to (2) that the relevant process (2.3) does not involve any further reasoning on $S$’s part. Of course $S$’s feeling compelled to accept $\varphi$ can be overdetermined; the above condition concerns one process that is causally sufficient for feeling compelled to accept $\varphi$.

For a definition of finding $\Delta \Rightarrow \varphi$ overtly compelling [overtly primitively compelling] by virtue of being an instance of a given rule, add to (2) that the relevant process (2.4) depends on $S$’s sensitivity to the fact that $\Delta \Rightarrow \varphi$ is an instance of that rule. (This idea is Peacocke’s response to “Kripkestein’s” worries; see previous citation.)

The corresponding notion for multiple-conclusion formula-inferences involves rejection as well as acceptance; I’ll set it aside for this paper. (The key idea for $\Delta \Rightarrow \Gamma$: given that $S$ accepts $\Delta$, $S$ would feel compelled not to reject all members of $\Gamma$.)

This is only a first try, at least if $L$ is a social language rather than an ideolect. A fuller characterization will also consider $S$’s dispositions to accept corrections, and recognize other’s errors, with regard to what inferences $S$ accepts, where activation of these dispositions also involves sensitivity to the inferences being instances of given rules.

Conceive of acceptance occurring in a specious present in which the subject
can accept every member, keeping them all “in mind”, with no shift of context.
What if \( \Delta \) is large? Then simultaneous acceptance might be impossible for \( S \), as
\( S \) actually is; for example, \( S \)'s brain might not be big enough. No matter: \( S \) could
have been in the triggering-condition if \( S \) were significantly different; we needn’t
demand that it be feasible for the \( S \) to accept \( \Delta \). (“Kripkestein” might object:
we’d have no idea what \( S \) would do if \( S \) were so different from the way \( S \) actually
is that \( S \) could accept a large \( \Delta \). I disagree: extrapolating from what \( S \) does when
accepting small \( \Delta \)s gives us some basis on which to form rational beliefs about
what \( S \) would do if \( S \) were to accept a large \( \Delta \). Be that as it may, the force of the
objection is not completely clear if one doesn’t buy an analysis of dispositions in
terms of conditionals.\(^4\)

Also, keep in mind that manifestation of a disposition can be blocked: all sorts
of psychological factors may obstruct \( S \)'s feeling compelled to accept \( \varphi \). In many
such cases \( S \) would at least experience cognitive dissonance. Furthermore, \( S \) may
feel compelled to accept \( \varphi \) and still not do so. Does this account imply that if \( S \)
grasps the sense of \( L \)'s logical constants, \( S \) will be disposed to feel compelled to
accept the conclusion of any complicated deductively correct argument given that
\( S \) accepts its premises? No. Suppose \( S \) is disposed to feel compelled to accept
\( \varphi_1 \) conditionally on accepting \( \varphi_0 \), and is disposed to feel compelled to accept \( \varphi_2 \)
conditionally on accepting \( \varphi_1 \). \( S \) needn’t be disposed to feel compelled to accept
\( \varphi_2 \) given that \( S \) accepts \( \varphi_0 \). Suppose \( S \) does accept \( \varphi_0 \), and so feels compelled
to accept \( \varphi_1 \); \( S \) might not give in to that feeling, and so not trigger the second
disposition, and so not feel compelled to accept \( \varphi_2 \). Or perhaps \( S \) does accept \( \varphi_1 \),
but that somehow destroys the second disposition. Or perhaps it merely weakens
it, so that \( S \) is disposed feel compelled to accept \( \varphi_2 \) given that \( S \) accepts \( \varphi_0 \), but
that disposition is significantly weaker than the two first-mentioned dispositions;
in that case a sufficiently longer chain might not be associated with a disposition
of \( S \) to feel compelled to accept some \( \varphi_n \) given that \( S \) accepts \( \varphi_0 \).

Now for a look at acceptance. At its most straightforward, acceptance is an
attitude towards statements in a given language, where a statement is a sentence,
and so a syntactic object, supplemented with a “reading”, i.e. disambiguated and
with indexical parameters tied to appropriate contextually determined values.
(Thus a statement has its truth-conditions necessarily.) Of course acceptance is
relative to a language. As I here understand it, acceptance is not a propositional
attitude, since propositions are not syntactic objects. When one believes the
content of a statement – the proposition it expresses, what it “says” – one accepts

\(^4\)This is developed in [Michael Fara, unpublished].
that statement. But there is reason to allow for accepting formulas with free variables. (A formula is usually understood to be an “open sentence”, i.e. either a sentence or the result replacing some occurrences of constants in a sentence by free occurrences of variables of appropriate type. I’ll understand a formula to be an “open statement”, i.e. either a statement or the result of carrying out such replacements on a statement.) In thinking through an argument formalized as a Natural Deduction derivation, one might accept a formula \( \varphi \) containing free occurrences of variables (what some call “parameters”) that are not assigned any values; in this case \( \varphi \) does not express a proposition. We frequently pretend that we have been “given”, or have ourselves “fixed”, values for variables occurring free in \( \varphi \); but this is heuristic patter. (I reject the thesis that every entry in an argument expresses a proposition; the entries with free variables merely express conditions.) So in full generality, acceptance is an attitude towards formulas.

Acceptance is a cognitive, not a behavioral, relation. Think of accepting \( \varphi \) as consisting in an act of comprehending \( \varphi \), as a formula of \( L \), that elicits an act of inward – and perhaps also outward – affirmation directed towards \( \varphi \). Suppose that this is unproblematic for atomic formulas. We want to use the notion of acceptance of formulas of \( L \), some of which contain occurrences of \( c \), to characterize grasping the sense of a logical constant \( c \) in \( L \). So grasp of \( c \)’s sense in \( L \) is tied by a “local holism” to grasp of a range of formulas of \( L \). And, if \( c \) isn’t \( L \)’s only logical constant, some of the relevant formulas contain other logical constants; so this “local holism” involves the grasp of the senses of all of \( L \)’s other logical constants. To show that this isn’t a vicious circularity, we need to Ramsify. The details that follow are a bit digressive; the impatient reader may skip ahead to the last paragraph of the following section.

§3. First, let’s suppose that \( c \) is the only logical constant in \( L \). Suppose that subject \( S \) grasps the sense of formula \( \varphi \), which I’ll abbreviate as “s-grasps \( \varphi \)”. This is to say that \( S \) is in a standing mental state \( s \), \( s = s \)-grasp of \( \varphi \), that’s relational with respect to \( \varphi \), and perhaps with respect to other things as well. I take it that \( s \) either is or is constituted by \( S \) being in a bunch of substates, themselves standing mental states of \( S \), and that among them is \( S \)’s \( s \)-grasp of each constituent of \( \varphi \); if \( c \) is a constituent of \( \varphi \), \( s \)-grasp of \( c \) is a substate of \( s \). Let \( p \) be the psychological process-type whose tokens in \( S \) would consist of \( S \)’s thinking of or perceiving \( \varphi \), that event interacting with \( s \), leading \( S \) to regard \( \varphi \) with inner affirmation. (In this process, \( S \) enters the occurrent state of comprehending \( \varphi \).) Let ‘\( M(x, c) \)’ abbreviate ‘\( x \) is a mental state relational with respect to \( c \)’. So certainly \( M(s \)-grasp of \( c, c \)).
Assume that \( M(x, c) \); we’ll now define a state \( s(x, \varphi) \) and then relations \( G(x) \) and \( A(x) \) that might hold between a subject \( S \) and formula \( \varphi \). Let \( s(x, \varphi) \) be the state that we’d obtain by taking \( s \) and replacing s-grasp of \( c \) by \( x \); so \( S \) would be in \( s(x, \varphi) \) if \( S \) were in a standing mental state as much like being in \( s \) as is nomologically possible, except that \( S \) is in \( x \) rather than s-grasping \( c \). (If \( x \) is the state of grasping an alternative sense that \( c \) might have had, then \( s(x, \varphi) \) is a state of grasping a sense that \( \varphi \) might have had. But if \( x \) is not a state of the former sort, \( s(x, \varphi) \) is not a state of the latter sort; in general, \( s(x, \varphi) \) may be a state of no psychological interest, one in which \( x \) interacts in no interesting ways with s-grasp of the constituents of \( \varphi \) other than \( c \).) So \( s(x, \varphi) \) is relational with respect to \( \varphi \), and in particular, \( s = s(s\text{-grasp of } c, \varphi) \). Let \( p(x, \varphi) \) be the process we’d get by taking \( p \) and replacing s-grasp of \( c \) by \( x \); so \( S \) would undergo \( p(x, \varphi) \) if \( S \) underwent a process as much like \( p \) as is nomologically possible, except that \( S \) is in \( x \) rather than s-grasping \( c \). (If \( x \) is the state of grasping an alternative sense for \( c \), \( p(x, \varphi) \) would terminate with \( S \) regarding \( \varphi \) with inward affirmation; otherwise, probably \( p(x, \varphi) \) wouldn’t be a coherent process at all.) So \( p = p(s\text{-grasp of } c, \varphi) \).

With \( s(x, \varphi) \) and \( p(x, \varphi) \) specified, let’s drop the assumption that \( S \) s-grasps \( \varphi \). Let \( S \) bear \( G(x) \) to \( \varphi \) iff \( S \) is in state \( s(x, \varphi) \). In particular, the relation of s-grasping between subjects and formulas of \( L \) is the relation \( G(s\text{-grasp of } c) \). For \( S \) to bear \( G(x) \) to \( c(\psi, \theta)^\frown \) would be for \( S \) to bear \( G(x) \) to both \( \psi \) and \( \theta \), to be in state \( x \), and for these three states to be as close to being inter-related appropriately – in whatever way \( S \)'s s-grasp of \( \psi \) and \( \theta \) would be inter-related to \( S \)'s s-grasp of \( c \) were \( S \) to s-grasp \( c(\psi, \theta)^\frown \). Define \( A(x) \) similarly, so that the relation of acceptance between subjects and formulas of \( L \) is the relation \( A(s\text{-grasp of } c) \). E.g. suppose that \( \psi \) and \( \theta \) are atomic formulas and \( c \) is a 2-place connective. For \( S \) to bear \( A(x) \) to \( c(\psi, \theta)^\frown \) would be for \( S \) to bear \( G(x) \) to \( c(\psi, \theta)^\frown \), to think of or perceive \( c(\psi, \theta)^\frown \), and for the former states to interact with the latter event so as to initiate \( p(x, c(\psi, \theta)^\frown) \).

Continuing under the assumption that \( M(x, c) \), we now must define \( S \)'s bearing \( FOC(x) \) to \( \Delta \Rightarrow \varphi \). The idea is that bearing \( FOC(x) \) to a formula-inference is to Finding it Overly Compelling as \( G(x) \) and \( A(x) \) are to s-grasping and acceptance. For \( S \) to bear \( FOC(x) \) to \( \Delta \Rightarrow \varphi \) is: (1) for \( S \) to be disposed to feel compelled to bear \( A(x) \) to \( \varphi \) given that \( S \) bears \( A(x) \) to \( \Delta \), and (2) if \( \varphi \notin \Delta \), for that feeling to be brought about by a process (2.1) initiated at most by \( S \)'s bearing \( A(x) \) to \( \Delta \) and \( S \)'s bearing \( G(x) \) to \( \varphi \), and (2.2) not depending on \( S \)'s prior bearing of \( A(x) \) to \( \varphi \). So finding \( \Delta \Rightarrow \varphi \) overtly compelling is bearing \( FOC(s\text{-grasp of} \)
c) to \( \Delta \Rightarrow \varphi \). To obtain a definition of bearing \( FOPC(x) \) to \( \Delta \Rightarrow \varphi \), this the analog with free-\( x \) of Finding it Overtly Primitively Compelling, add clause (2.3), requiring the process not to involve further reasoning. Similarly, to define bearing \( FOC(x) \), or \( FOPC(x) \), to \( \Delta \Rightarrow \varphi \) by virtue of its being an instance of a rule, add clause (2.4).

Finally, suppose that \( \mathcal{R} \) and \( \mathcal{R}_0 \) are as above. Fully grasping \( c \)'s sense in \( L \) is that mental state \( x \) such that (1) \( M(x, c) \), and (2) \( x \) would, under normal conditions, dispose any subject in \( x \) to bear \( FOPC(x) \) to instances of members of \( \mathcal{R} \), this by virtue of their being instances of those rules. Adequately grasping \( c \)'s sense in \( L \) is that mental state such that (1) \( M(x, c) \), and (2) \( x \) would, under normal conditions, dispose any subject \( S \) in \( x \) to bear \( FOPC(x) \) to instances of members of \( \mathcal{R}_0 \), this by virtue of their being instances of those rules, and (3) \( x \) would under normal learning conditions dispose \( S \) to learn to bear \( FOPC(x) \) to instances of members of \( \mathcal{R}_1 \), that by virtue of their being instances of those rules, so that that learning doesn’t involve adding a homonym to \( S \)'s lexicon.

If \( L \) contains other logical constants \( d \), etc., we need to revise the above remarks as follows. Assume that \( M(y, d) \), ... In place of the state \( s(x, \varphi) \) and relations \( G(x) \) and \( A(x) \), we must define the state \( s(x, y, ..., \varphi) \), the process-type \( p(x, y, ..., \varphi) \) and the relations \( G(x, y, ...) \), \( A(x, y, ...) \), and then \( FOPC(x, y, ...) \). Then we add existential quantifications to the preceding condition thus: full grasp of \( c \)'s sense in \( L \) is the mental state \( x \) such that: (1) \( M(x, c) \), and (2) for some mental states \( y, ..., M(y, d) \) and ..., and \( x \) would, under normal conditions, dispose any subject \( S \) in \( x \) to bear \( FOPC(x, y, ...) \) to instances of members of \( \mathcal{R} \), this by virtue of their being instances of those rules. A similar supplement applies to adequately grasping \( c \)'s sense.

What if more than one \( x \) meet this condition (for full grasp or for adequate grasp)? I take it that a mental state is individuated by its functional role in a subject’s psychology; hopefully our condition specifies such a role. If it appears that two distinct states satisfy that condition, that shows that they were not individuated at the right “grain”, and are merely two ways in which a single mental state gets realized.

Obviously it’s easier to think about all this in terms of grasping \( c \)'s sense and acceptance rather than in Ramsified terms; so I’ll stick to that easier vocabulary.

So much for formula-inferences; now for sequent inferences. For this, think of finding a formula-inference (primitively) compelling as itself a kind of acceptance: when \( S \) finds a formula-inference \( \Delta \Rightarrow \varphi \) (primitively) compelling, let’s say that \( S \) (primitively) \( c \)-accepts \( \Delta \Rightarrow \varphi \) (‘\( c \)' for ‘compelling’). For \( S \) to find the sequent
inference \(\langle D, \Delta \Rightarrow \varphi \rangle\) compelling by virtue by virtue of its being an instance of a rule is for (1) \(S\) to be disposed to feel compelled to accept \(\varphi\) given that \(S\) accepts \(\Delta\) and \(c\)-accepts all members of \(D\), and (2) if \(\varphi \notin \Delta\), for that acceptance of \(\varphi\) to be brought about by a process (2.1) initiated by \(S\)’s acceptance of \(\Delta\), \(S\)’s \(c\)-acceptance of the members of \(D\), and \(S\)’s grasp of \(\varphi\)’s sense, (2.2) not depending on \(S\)’s prior acceptance of \(\varphi\), (2.3) involving her sensitivity to the fact that \(\langle D, \Delta \Rightarrow \varphi \rangle\) is an instance of that rule, and (2.4) not involving any further reasoning on \(S\)’s part. (Does this amount to the following: “\(S\) is disposed to \(c\)-accept \(\Delta \Rightarrow \varphi\) given that \(S\) \(c\)-accepts all members of \(D\), and for ...”? I’m not sure, but doubt it. There seems to be a difference between (1) being disposed to \(\gamma\) given \(\alpha\) and \(\beta\), and (2) being disposed to (be disposed to \(\gamma\) given \(\alpha\)) given \(\beta\).)

§4. Thesis 1 says that sense-constituting rules for logical constants are syntactic, making no direct reference to referential or pragmatic relations. If ‘semantics’ stands for the study of linguistic understanding, rather than the theory of reference and truth, semantics for logical constants is syntactic. A logical constant has its semantic value because of the sense-constitutive rules that govern it, not the converse. My slogan for logic is “Syntax first”. “Syntax first” is suggested by remarks of Wittgenstein, Carnap, Gentzen and Popper\(^5\); Kneale came closest to clearly advocating it: “... formal (or logical) signs are those whose full sense can be given by laying down rules of development for the propositions expressed by their help.”\(^6\) More recently, Powers\(^7\) and Hacking\(^8\) have advocated it.

Carnap went wrong in claiming that any set of rules concerning an expressions role in argumentation could constitute a sense for that expression: “... let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols.”\(^9\) Dummett seems to think that Carnap’s claim “would necessarily be so” if Thesis 2 were true.\(^10\) He says: “... if a grasp of the meaning of a logical constant consisted solely in a readiness to acknowledge as correct those inferences involving it which exemplified one of the rules in some suitable basic set of such rules”, then “any arbitrary (consistent) set of rules of inference admits

\(^5\)See [Karl Popper, ’47].
\(^6\)See [William Kneale, ’56], pp. 254-5. Kneale’s rules of development are rules of multiple-conclusion reasoning; in this his proposal differs from mine.
\(^7\)See [Lawrence H. Powers, ’78].
\(^8\)See [Ian Hacking, ’79].
\(^9\)See the Foreword to [Rudolf Carnap, ’34].
\(^10\)See [Michael Dummett, ’77], p. 362.
a range ... of meanings for the logical constants involved under which those and only those rules of inference that are derivable from that set are valid”. He gives no argument for this strong claim, which I think false. Not just any rules, or even just any introduction and elimination rules for a constant can be constitutive of sense; this is a lesson to be learned from Prior’s ‘tonk’.11 (Here I assume that ‘tonk’ does not express a sense. Perhaps we could as well say that it expresses a defective sense, just as we might take ‘true in English’, as naively understood, to express a sense – an incoherent concept that can lead those who possess it into inconsistency. Does anything hang on which we say? I’m not sure.) I’ll come to the question of what sets of rules are sense-constituting in §9.

Gentzen went wrong in suggesting that, for all the logical constants he discussed, introduction rules have meaning-determining priority over elimination rules. At least I know of no adequate explication of this supposed priority. There is a respect in which the introduction rules for some logical constants, e.g. expressions of Negation, Disjunction and first-order Existence, are cognitively prior to their elimination rules: the former rules overtly govern, and overtly constitute the senses of, such expressions, while for many competent speakers the latter rules only tacitly govern such expressions. But the reverse holds for others logical constants, e.g. expressions of Material Conditionality and first-order Universality. Expressions of Conjunction are rather special: for them, there is no such priority either way; all constituting rules are overtly constituting, and ordinary speakers fully grasp the sense of such expressions; ‘and’ is easy. I’ll return to this in §5.

Dummett thinks that Thesis 1’ requires that “the condition for the correctness of an assertion made by means of a sentence containing a logical constant must always coincide with the existence of a deduction, by means of those [sense-constituting] rules to that sentence from correct premisses none of which contains any ... logical constants.”12 Peacocke thinks that the concepts Conjunction and Universal Quantification over the Natural Numbers are constituted by deductive rules.13 But he goes along with Dummett’s requirement; this leads him to deny that the concept of Negation is constituted by deductive rules, maintaining that it’s constituted by a broader class of rules that he calls ‘transitional’ rules.14 I see no reason to accept Dummett’s remarkably strong requirement. (He provides no argument for it in the text cited.) Of course I reject Peacocke’s doctrine about

11See [Arthur Prior, ’60].
12See [Michael Dummett, ’77], p. 363.
13See [Christoper Peacocke, ’92], pp. 6-7.
14See [Christoper Peacocke, ’86], pp. 91-2.
Negation.

Logical constants have their truth-relevant properties, including their “semantic values” (following Dummett’s Fregean approach to semantic theorizing), because of their roles in argument, not vice-versa. This ‘because’ means “in part because”: certain constraints on truth and the like will matter as well; see §10. I reject the neo-Davidsonian doctrine according to which for a subject S to grasp the sense of an expression of Conjunction in L, let’s say by ‘&’, is for S to know (or if you prefer, cognize) that for any statements φ and ψ of L, \[ \varphi \land \psi \] is true in L iff \( \varphi \) is true in \( L \) and \( \psi \) is true in \( L \) (or more generally the corresponding conditions for satisfaction of formulas). This doctrine seems to imply the following: for a young child to come to understand ‘and’ in English, she or he first needs to bear some cognitively significant relation to the property of being a true statement, or perhaps utterance, in English (perhaps under a mode-of-presentation of English as “the language spoken around me”), as well as to Material Biconditionality, and to Universality restricted to statements, or utterances, in English. Perhaps this can be less “developed” than possession of a concept of being a true statement or utterance in English, of Material Biconditionality, etc.; this is the point of the fudge-word ‘cognize’. But even this seems to ask a lot of an infant learning English – too much, in my opinion.

Davidson himself has been careful to avoid making such a substantive claim about actual linguistic understanding. According to Davidson, the right sort of semantic theory of \( L \) is at least part of “what must be said to give a satisfactory description of the competence of the interpreter”; this implies that “some mechanism in the interpreter must correspond to the theory.”\(^{15}\) This second claim, whatever it comes to, seems consistent with “Syntax first”. The first claim raises the question of whether one “must say” the important things supported by other things that one “must say”. I’ve suggested that a satisfactory description of the competence of an understander (that is, a Davidsonian interpreter) requires that we attribute dispositions to conditional feelings of compelled acceptance. These facts at the level of sense have important consequences at the level of reference, the level described by a Davidsonian semantic theory. Davidson’s first claim is true of such a theory if a satisfying theoretical description of linguistic understanding must spell out these consequences about reference.

Let me digress to extend “Syntax first” from logical concepts to our concepts of logical consequence and logical entailment: our “original” concepts of these relations are also syntactic. In so far as the person-in-the-street has a concept of

\(^{15}\)From [Donald Davidson, ’85], p. 469.
logical entailment, it’s the concept of the existence of a syntactic object: a demonstrative argument – one such that each inference in it one would find compelling – from premises to a conclusion. I don’t deny that by the 19th century a semantic conception of logical entailment was in circulation among philosophers. But this was the product of proto-mathematical discovery, proto-mathematical in that it looked forward to rigorous semantic definitions (most importantly, the standard model-theoretic definition) for formal languages that crystallized in Tarski’s wake; this was an informative reconception of logical entailment, not the result of mere conceptual analysis. As for the informal, so for the rigorous: the relation between derivability in a Natural Deduction formalization of classical first-order logic and any of several semantic definitions of classical first-order consequence is like that between a formulation of nominal essence, or of the reference-fixing description on which we originally rely in our referential access to a natural kind, and a formulation of its real essence, e.g. between specifying the perceptual and operational properties by which people first fixed the reference of ‘gold’ and saying that gold is stuff whose atoms each contain 79 protons. (For evidence that early in his career Russell thought of logical entailment syntactically, at least when he thought of it at all, see [Ian Proops, 2002].\textsuperscript{16} There Proops also discusses a passage in which Frege characterizes what it is for a thought to “be dependent on” a group of thoughts in terms of an iteration of making logical inferences; though not explicitly syntactic, the reliance on recursion suggests that he too was thinking of this syntactically.)

§5. To my knowledge, the literature in logic on rules only considers rules governing particular languages. But it’s important to conceive of a deductive rule, and with it of a logical concept, as a language-transcendent object. (This is especially important for variable-binding logical constants, e.g. expressions of quantification. For example, when we introduce a new name, we replace our language by an expanded language, one that includes new instances of Universal introduction; we don’t want to have to say that we’ve adopt a new rule of Universal introduction.) A logical rule is realized in \(L\) by the set of its instances in \(L\); \(L\)’s assignment of logical constants to their senses and \(L\)’s argument-conditions determine these realizations. A word on argument-conditions.

To specify a language \(L\) as a formal object, we need to specify the class of deductive arguments \(L\) allows. This involves specifying the overall structure of such arguments, for example whether \(L\) allows for multiple-conclusion formula-\textsuperscript{16} Which discusses [Bertrand Russell, ’94], pp. 513-5, especially paragraph 2 on p. 515.
inferences. For this paper, this is all that matters regarding $L$’s argument-conditions. But let me digress a little. Argument-conditions also constrain an aspect of argument that’s something like mood, what I’ll call “mode of acceptance”. As far as this paper is concerned, to accept a statement is to accept it as actually true – this is the primary mode of acceptance. But in making suppositions we can also accept a statement as true relative to non-actual possibilities. If bivalence fails, one might accept a statement as non-true rather than as true, either actually or relative to non-actual possibilities. An adequate understanding of intensional logical constants and multi-valued reasoning (which I think we do, though badly) will require considering multi-modal inferences.\footnote{Consider bi-modal arguments in [Harold Hodes, ’86], [Harold Hodes, ’87], and [Harold Hodes, ’89].} For this paper we’ll confine our attention to the primary mode of acceptance.

Some examples should help.\footnote{And serve as an advertisement for my unpublished work on universal proof-theory, in which I develop more fully some of the ideas touched on in this paper.} Let’s suppose that $L$ allows only for single-conclusion formula-inferences, and that $L$’s lexicon contains familiar constants; we’ll consider some well-known deductive rules, each involving only a single logical constant. The realization of Conjunction Introduction in $L$, $\&$-$\text{Intr}_L$, is the set of sequent-inferences of this form:

$$\langle \{ \Delta_i \Rightarrow \psi_i : i = 0, 1 \}; \Delta_0, \Delta_1 \Rightarrow (\psi_0 \& \psi_1)^\gamma \rangle,$$

for any $\Delta_i \subseteq \text{Sent}(L)$, $\psi_i \in \text{Sent}(L)$, $i = 0, 1$. Similarly, the realization of Conjunction Elimination in $L$, $\&$-$\text{Elim}_L$, is the set of sequent inferences of this form:

$$\langle \{ \Delta; \psi_0, \psi_1 \Rightarrow \psi_2 \}; \Delta, (\psi_0 \& \psi_1)^\gamma \Rightarrow \psi_2 \rangle.$$

The realization of Conditional Introduction in $L$, $\Rightarrow$-$\text{Intr}_L$, is the set of sequent inferences of this form:

$$\langle \{ \Delta, \psi \Rightarrow \theta \}; \Delta \Rightarrow (\psi \Rightarrow \theta)^\gamma \rangle.$$

The realization of Disjunction Elimination in $L$, $\lor$-$\text{Elim}_L$, is the set of sequent inferences of this form:

$$\langle \{ \Delta_0, \psi_0 \Rightarrow \varphi; \Delta_1, \psi_1 \Rightarrow \varphi \}; \Delta_0, \Delta_1, (\psi_0 \lor \psi_1)^\gamma \Rightarrow \varphi \rangle.$$
natural numbers 0 and 1 to represent first-place and second-place for any binary formula connective, and 2 represents a place for the consequent of the conclusion of an elimination-rule for such a connective. In what follows, ‘/0’ is a notation for \( \langle \{\}, 0 \rangle \), ‘0, 1/2’ for \( \langle \{0, 1\}, 2 \rangle \), etc. We may represent the language-transcendent rules just considered thus: Conjunction Introduction = \{/0; /1\}; Conjunction Elimination = \{0, 1/2\}; Conditional Introduction = \{0/1\}; Disjunction Elimination = \{0/2; 1/2\}.

A logical concept, the sense of a “possible logical constant”, is also language-transcendent. In accord with Thesis 1′, I suggest that a logical concept is also a mathematical object, one composed, so to speak, of deductive rules. For a constant of \( L \) to express a logical concept is for the rules making up that concept to constitute that constant’s sense in \( L \) (with that construed in terms of overt and tacit primitive governance for \( L \)-understanders). The lexicon of a language \( L \) assigns each logical constant \( c \) to a logical concept, and thus to deductive rules \( \mathcal{R} \), or better \( \langle \mathcal{R}_0, \mathcal{R}_1 \rangle \). The rest of \( L \)'s lexicon and \( L \)'s formation-rules then determine the realizations for \( L \) of the rules in \( \mathcal{R} \). And now I’m ready to propose:

Thesis 3. Only rules that are, broadly speaking, introduction rules and elimination rules can constitute the sense of a logical constant. (I say broadly speaking because I don’t know of any fully general characterization of what should count as an introduction or an elimination rule.\(^{19}\))

The familiar introduction and elimination rules sit in a natural hierarchy, one that generates a corresponding hierarchy of logical concepts that involve those rules, and thus of corresponding logical constants. The rules of level 0 are distinguished by their “separability”: each concerns a single occurrence of a single constant, the main constant of an instance’s “main” or “principle” formula. The Big Five connective-concepts (Absurdity, Conjunction, Disjunction, Material Conditionality, Material Biconditionality) and the first-order Universality and Existence (as usually understood) are of level 0, this because their introduction and elimination rules are all of level 0. (Note: the usual rule for Surd is an elimination rule; Surd has no introduction rule.) Negation is intrinsically more complex than the Big Five; properly speaking, Negation introduction involves Surd; its realization in \( L \) is the set of sequent inferences of this form:

\[
\langle \{ \Delta, \psi_0 \Rightarrow \bot \} ; \, \Delta \Rightarrow \Gamma \Rightarrow \neg \psi_0 \Rightarrow \Gamma \rangle .
\]

It, along with Neither-nor and If-then-else, is of level 1. This step from level 0 to level 1 iterates, generating the mentioned hierarchy.

\(^{19}\)I take some steps in this direction in unpublished work on universal proof-theory.
Do all introduction and elimination rules sit in this hierarchy? Or can there be logical “local holisms”? What we make of free logics and singular existence-statements depends on this is a delicate and important question, which I’ll put aside. (A predicate of singular existence should, I think, count as a logical constant; and it has an introduction-rule that suits certain metaphysical tastes. But its elimination rules would seem to be exactly the introduction and elimination rules for expressions of first-order Existence and Universality in a free-logic. There is an interesting issue here.)

In addition to introduction and elimination rules, there are what I call “thickening” rules, rules that shed assumptions in formula-inferences. (So-called because adding assumptions to a formula-inference is sometimes called “thinning”.) Such a rule permits us to infer a formula-inference from formula-inferences with the same consequent whose antecedents include formulas not in the antecedent of the conclusion. Excluded Middle and Generalized Excluded Middle are thickening rules. Their realizations in $L$ are the sets of sequent inferences of these forms respectively:

\[
\begin{align*}
&\langle \{ \Delta_0, \neg \psi \vdash \varphi ; \; \Delta_1, \psi \vdash \varphi \} ; \; \Delta_0, \Delta_1 \Rightarrow \varphi \rangle , \\
&\langle \{ \Delta_0, \neg (\psi \supset \theta) \vdash \varphi ; \; \Delta_1, \psi \vdash \varphi \} ; \; \Delta_0, \Delta_1 \Rightarrow \varphi \rangle .
\end{align*}
\]

So members of EM$_L$ are members of GEM$_L$ with $\theta$ taken to be ‘$\bot$’. (Other thickening rules generate intermediate logics when added to intuitionistic logic.) One thickening rule is of great mathematical importance, but (to my knowledge) has received no attention. I call it the rule of Infinite Domains. Its realization in $L$ is the set sequent inferences of the following form: for set of formulas $\Delta$, any formula $\varphi$, any natural number $n$, any terms $\tau_0, \ldots, \tau_{n-1}$, and any variable $v$ not occurring free in any member of $\Delta$, in $\varphi$, or in any $\tau_{i \leq n}$:

\[
\langle \{ \Delta, \neg v = \tau_0 \vdash, \ldots, \neg v = \tau_{n-1} \vdash \Rightarrow \varphi \} ; \; \Delta \Rightarrow \varphi \rangle .
\]

The hierarchy of introduction and elimination rules extends to thickening rules: GEM is of level 0, since it concerns only expressions of Material Conditionality, which is of level 0; EM and ID are of level 1. (So GEM is more basic than EM; this should undercut the widespread idea that the fundamental proof-theoretic difference between intuitionistic and classical logic concerns Negation; rather it concerns Material Conditionality.)

Vague Conjecture 1: Once we have an adequate account of what introduction, elimination and thickening rules are, we’ll see that they suffice to uniquely characterize the role of a logical constant in demonstrative argumentation.
All the rules considered above are purely syntactic. What rules are not? An example: think of ‘true in English’ as a constant predicate. It is characterized by certain introduction and elimination rules; we may conceived of the realization of these rules in English as sets of sentence inferences with members like the following, where ‘a’ names ‘Snow is white’:

\[
\langle \{ \Delta \Rightarrow \text{‘Snow is white’} \} ; \Delta \Rightarrow \text{‘a is true-in-English’} \rangle ;
\]

\[
\langle \{ \Delta, \text{‘Snow is white’} \Rightarrow \theta \} ; \Delta, \text{‘a is true-in-English’} \Rightarrow \theta \rangle .
\]

In full generality, the realization of these rules in English lead to inconsistency: ‘true in English’ is a defective. Various ways of constructing consistent semantics for ‘true in English’ amount to proposals to replace it with a non-defective constant. Let’s pretend that we’ve done this, and the replacement is homonymous with ‘true in English’. The important point is this: these characterizing introduction and elimination rules are not purely syntactic; whether a sequent-inference is an instance of these rules depends on some semantic information. For the above example, we need to specify that ‘a’ designates ‘Snow is white’. ‘True in English’ is what I’ll call a semi-logical constant.

For any \( L \) that we can translate into English, we can introduce the predicates ‘true-in-\( L \)’ and ‘false-in-\( L \)’ into English, governed by corresponding introduction and elimination rules. If \( L \) is well-enough behaved, e.g. if it lacks semantic vocabulary, these constant predicates are not defective. The above point applies to satisfaction and frustration as well as to truth and falsity. For any formula \( \varphi \) of \( L \), let \( \text{trans}_{\varphi} \) be its translation into English. The introduction and elimination rules for \( \text{‘satisfies-in-} L \text{‘} \) has instances like these, for any variable assignment \( A \), \( \sigma \) a singular term in English referring to \( \varphi \), \( \alpha \) a singular term referring to \( A \), and \( \text{trans}_{\varphi}' \) formed by replacing each free occurrence of each variable \( v \) free in \( \text{trans}_{\varphi} \) by a fresh singular term designating \( A(v) \):

\[
\langle \Delta \Rightarrow \text{trans}'_{\varphi} ; \Delta \Rightarrow \text{‘\( \alpha \) satisfies-in-} L \text{‘} \sigma' \rangle ;
\]

\[
\langle \{ \Delta, \text{trans}'_{\varphi} \Rightarrow \theta \} ; \Delta, \text{‘\( \alpha \) satisfies-in-} L \text{‘} \sigma' \Rightarrow \theta \rangle .
\]

Again, these rules are not purely syntactic; in the generalization to satisfaction, we need to specify that \( \sigma \) and \( \alpha \) designates \( \varphi \) and \( A \) respectively. Predicates like ‘true-in-\( L \)’, ‘satisfies-in-\( L \)’, etc. are also semi-logical constants.

Note: if we restrict the introduction and elimination rules for ‘true-in-English’ and ‘false-in-English’ to instances in English, and require that the terms of which these predicates are predicated in these instances be quote-names, we’d obtain
purely syntactic rules, since we could state these restricted rules without attaching riders like "‘a’ designates ‘Snow is white’". From this one might conclude that in a way ‘true-in-English’ and ‘false-in-English’ are logical constants after all. But this is an illusion. Such rules would not constitute the sense of the predicate ‘true-in-English’; they would constitute the sense of a connective written in an odd way (attaching to a sentence by prefixing that sentence with a left-single quote-mark, and appending it with a right-single quote-mark followed by ‘is true-in-English’). This connective would express the 1-place redundant operator, with the introduction rule \{/0\} and the elimination rule \{0/1\}. There is no purely syntactic way to make quote-names of sentences be singular terms designating the sentences within the quotes.

Thesis 4. Semi-logical constants of a language constitute a natural, though quite small, semantic kind; their senses are constituted at least in part by partially semantic introduction and elimination rules. These are all predicates.

We could replace the introduction and elimination rules for \text{true-in-}L\text{true-in-}L and \text{false-in-}L\text{false-in-}L by the instances of Tarski’s Schema Tr and the corresponding Schema Fa, the latter with instances like this:

\[ b \text{ is false-in-English iff snow isn’t white,} \]

where ‘b’ designates ‘Snow is white’. I think the rules are more fundamental: these rules could govern \text{true in } L\text{true in } L even in a meta-language so impoverished that it had no way to express Material Conditionality or Biconditionality. But we need the schematized biconditionals if we prefer theories in which all theorems are provable by purely syntactic rules – rules of logic properly so-called. This preference is widespread and understandable. Later it will be useful to have on our table the following schemata for satisfaction and frustration, corresponding to schemata Tr and Fa. For \( A, \alpha, \varphi, \sigma \) and \( \text{trans}_{\varphi}^\prime \) as above:

\[
\begin{align*}
\text{(Sat)} & \quad \alpha \text{ satisfies-in-}L \text{ } \sigma \text{ iff } \text{trans}_{\varphi}^\prime; \\
\text{(Fr)} & \quad \alpha \text{ frustrates-in-}L \text{ } \sigma \text{ iff it’s not the case that } \text{trans}_{\varphi}^\prime.
\end{align*}
\]

§6. The literature with which I’m acquainted identifies a logic with a theory in a particular language, one closed under a generous sort of substitution, or (marginally better) a similarly closed consequence relation on a particular language. This won’t do. As with rules, we need a language-transcendent conception of a logic. A logic is a four-tuple: (1) a set of types for lexical categories, e.g. the types
Formula, Individual Constant, Individual Variable, n-place Predicate Constant, n-place Formulas-to-Formula Operator that doesn’t bind variables (i.e. connectives), or that do (e.g. quantifiers); (2) a set of argument-conditions; details would take us far afield, but suffice to say that this component will determine whether the logic allows multiple-conclusion inferences; (3) a set of logical concepts, each of a unique type such that it would make sense for a logical constant of that type to express that concept; (4) a perhaps empty set of additional rules involving only logical concepts in the third set.

A language $L$ realizes a logic $L$ iff (1) the types in $L$’s first component correspond to non-empty lexical categories of $L$, (2) $L$ has argument-conditions that accord with $L$’s second component, (3) $L$ has logical constants of the appropriate categories that express the concepts in $L$’s third component, and (4) the additional rules in $L$’s fourth component govern the logical constants expressing the logical concepts involved in those rules. $L$ determines the set of provable formula-inferences in $L$, provable using only the rules provided by the third and fourth components of $L$. Such proofs can be “formatted” in a sequent-calculus or a natural deduction system; at this level of abstraction, a logic is neutral between such formats.

Setting aside issues of vagueness, I propose that $L$ realizes a unique “basic” logic, whose concepts are exactly those expressed by $L$’s logical constants and whose fourth component is empty: so all the rules built into $L$’s basic logic are sense-constitutive. $L$ also realizes a unique “total” logic, obtained from the basic logic by adding to its empty fourth component all the other rules primitively governing $L$’s logical constants. I’ve conjectured that these are all thickening rules.

One might object that a language need not have one total logic, since different kinds of discourse in it might be subject to different rules. Perhaps an English-speaking mathematician does constructive mathematics during the week and relaxes by doing classical mathematics on weekends. The objection is well-taken (assuming that English allows only for single-conclusion inferences); strictly speaking, it is a practice or type of discourse that realizes a logic. For convenience, I’ll retreat to a technical notion of languagehood, according to which our mathematician does his work in Constructive Mathematical English during the week but in Classical Mathematical English on weekends. The basic logics for Constructive and for Classical Mathematical English are identical; for most purposes we can take it to be standard first-order intuitionsitic logic. The total logic for

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20I address this in some unpublished work on universal proof-theory.
Constructive Mathematical English is obtained from its basic logic by adding at least the rule of Infinite Domains to its fourth component. The total logic for Classical Mathematical English is obtained by also adding EM or GEM.

Concepts of truth and falsity for statements are, of course, language-relative. I’ve built a logical practice into the identity of a language: we might have two languages that differ merely in whether their logical practices (viz. their total logics) are constructive or classical; for example, Constructive English and Classical English. This opens room for a distinction between concepts of constructive and classical truth for statements belonging to both languages. Note that this is not a distinction between different conceptions, or better different philosophical theories, of truth.

§7. Whether a purely syntactic rule overtly governs a constant in \( L \) is a matter of \( L \)’s syntax. This enlarges the scope of syntax in three respects. First: it concerns the syntactic structure of arguments, rather than merely that of single sentences or formulas. Secondly, whether a deductive rule govern certain expressions is a conditional matter, concerning conditional feelings of compelled acceptance. Grammaticality of sentences lacks this conditional structure. Thirdly, whether a rule overtly governs a logical constant for a subject \( S \) involves facts about \( S \)’s dispositions to accept statements, and those facts involve facts about \( S \) understanding of such statements. In contrast, it’s been claimed that whether a string of phonemes is grammatical in \( L \) (or \( S \)’s ideolect) involves only facts to which \( S \)’s understanding of that string is irrelevant.

In spite of these differences, there are continuities between argumentative syntax and the linguist’s “sentential” syntax. For one thing, the last claim might suggest that a native speaker classifies a string of phonemes as grammatical “directly” from its phonological properties. But no one does that; for most strings, a speaker – or better, a speaker’s “understanding module” – must first parse it into recognized words and assign these words to grammatical categories. These processes do not require sense-grasping, but they do bring the speaker’s lexicon into play. So the third gap between argumentative and sentential syntax is not as deep as it might initially seem.

Nor is the difference between the kinds of evidence at issue all that deep. The syntactician’s most basic evidence about which strings of phonemes in \( L \) are grammatical is information about which strings speakers of \( L \) produce and to which they respond. The syntactician in the field can get further evidence by querying a cooperative native, information about whether strings of phonemes “sound OK” to the native. We can’t expect speakers to have the concept of grammaticality at
“the personal level”, even if speakers’ language-processing modules might, in some sense, have this concept. Similarly the most basic evidence of what rules overtly govern $L$ is how speakers of $L$ reason in $L$, including what sort of criticism of reasoning they accept and give. Here the syntactician’s basic evidence is information about whether the natives actually accept particular formulas conditionally on their acceptance of particular sets of formulas; we can’t expect speakers to have the concept of logical entailment. Of course acceptance plays no role in the “sounds OK” response. But even here there are some commonalities. The logical syntactician will have to form hypotheses about whether responses occur because of sensitivity to structural properties of statements involved. The linguistic syntactician will have to form corresponding hypotheses about the “sounds OK” response: to assess whether an informant so responds merely because of sensitivity to the syntactic properties of a phonemic string, or because he or she understands what would be meant by someone who uttered that string (after all, we can understand a wide range of quite ungrammatical statements), or because he or she agrees with the thought the string expresses, or likes its prosodic features. One can’t avoid psychological hypothesis if one is to describe the sentential syntax or the argumentative syntax in play in a population.

To help fix ideas, consider a tribe that speaks a regimented first-order language $L$ of the sort beloved by logicians; and suppose its members engage in a significant amount of demonstrative argumentation – many of them are mathematicians – already formalized into standard first-order Intuitionistic Logic; and suppose that they are quite competent with all its rules. These are the sophisticated constructivists. Suppose a radical translator sets out to translate the logical constants of $L$. The syntax of $L$’s sentences will be easy to discern. The next step is to determine what rules overtly govern expressions of $L$. I suggest this: If our translator can tell when speakers make deductive inferences, can re-identify statements – or more generally, formulas –, can detect comprehension and acceptance reasonably well, and can form reasonable hypotheses about the psychological processes behind such responses, he or she has the ball rolling, even without any understanding of $L$ beyond that. In particular, I suggest that our translator will not need to translate any non-logical constants of $L$ in order to translate $L$’s logical constants (beyond those needed to detect comprehension and acceptance).

§8. By itself, Thesis 3 takes no position whether the basic logic for a language $L$ is classical or constructive. That depends on argument-formation in $L$, specifically on whether argumentative practice among speakers of $L$ involves only single-conclusion formula-inferences. I think that actual argumentative practice
among English speakers, in fact among all actual people, involves only single-conclusion formula-inferences, i.e. one argument condition of any natural human language is: each argument has a single-conclusion. We can represent classical reasoning as multiple-conclusion reasoning; but such representation is not a direct characterization of actual classical reasoning. (Keep in mind: multiple-conclusions are understood disjunctively.) If this psychological speculation is right, Thesis 3 implies that our basic logic is constructive.

This is not to commit myself to any so-called “anti-realist” theses, e.g. that truth is constituted by knowledge or justified belief, or that it a priori implies knowability. A mathematician might even believe that every proposition is either true or false, but still take no interest in classical mathematics because it is insufficiently computationally informative. (So I reject Tennant’s objection to M-realism: “One cannot simply give up the classical rules and carry on thinking like a realist. McDowell has failed to appreciate just what is involved, by way of semantic and philosophical foundations, in being an intuitionistic logician.”) I have no objection to classical logic, even though it’s not our basic logic.

Thesis 5. (1) The distinction between constructive and classical argumentation originates from a distinction between a more and a less demanding standard for reasonable belief for disjunctive and existential statements. (2) No logical constant is ambiguous between a constructive and a classical sense. (Well, at least not in the way many have supposed; e.g. expressions of Negation are not ambiguous in this way. In a bimodal logic accommodating truth-value gaps, there is room for a kind of disjunction that forms a truth even though the disjuncts lack truth-value, and room for a kind that can’t. It would seem appropriate to call the former “non-constructive” and the latter “constructive”. Perhaps ‘or’ in English is ambiguous between such connectives, e.g. in statements about future contingencies.)

That distinction between standards mentioned in (1) leads to a distinction between standards for non-conditional acceptance. If you want intentional contents for the assertions of mathematicians (and unless you’ve carried through the eliminativist’s project, you probably do), this also leads to a distinction between the proposition that a given statement constructively expresses and the one it classically expresses.

I actually have an argument for part (2). Things are clearest regarding the material conditional. Suppose we have two expressions, $\mathcal{C}_I$ and $\mathcal{C}_K$, the first with the constructive sense for the material conditional, the second with the purported classical sense. So $\mathcal{C}_I$ is governed by $\mathcal{C}_I$-Intr and $\mathcal{C}_I$-Elim, and $\mathcal{C}_K$ governed by

\footnote{See [Neil Tennant, 2002], p. 239.}
$\exists_K$-Intr, $\exists_K$-Elim and GEM. It’s easy to see that then $\exists_I$ is also governed by GEM. One might object that this merely shows that the constructive and classical senses for expressions of Material Conditionality can’t live in the same language. But if there really are two such senses, and we assign a distinct expression to each, how could that be impossible? It might be urged that there’s no possible language in which ‘water’ expresses it’s usual English-language sense and another word, say ‘twater’ expresses the sense ‘water’ expresses in twin-English. If this claim has any basis at all, it’s because grasping these senses would involve being in incompatible relations to external reality. But according to “Syntax first”, grasping the sense of a logical constant is a matter largely internal to the understander of $L$, the only external element being the expressions of $L$. Perhaps oil (or twater) and water can’t mix; but there’s no reason to think that distinct logical concepts can’t be expressed in a single language.

When Classical and Constructive mathematicians disagree, it can seem that they are really talking past each other: that what the Classical mathematician asserts on the basis of a non-constructive proof is not the proposition that the Constructive mathematician refuses to assert. This ecumenical content-pluralism should be appealing, at least to those who dislike disagreement. But – and this is the crucial point – classical content is not determined purely compositionally. The source of the misguided popular doctrine of the ambiguity of logical constants is blind faith in compositionality. Among us single-conclusion reasoners, constructive content is determined purely compositionally. But a speaker operating under a Classical logic makes assertions with classical content because at a second stage the logic kicks in, collapsing the constructive content to classical content.

One might think that if one is to use EM, or other rules that are not constitutive of the senses of the logical constants they govern, one needs a powerful justification. I think weak pragmatic justification suffices: such rules make mathematics easier. Be that as it may, gentlemen, and gentlewomen, prefer constructive proofs, this because they are more informative than proofs that make non-constructive inferences.

§9. What combinations of introduction and elimination rules can constitute the sense of a logical constant? And how does the part of the sense of a logical constant that a speaker adequately grasps determine the complete sense of that constant? According to “Syntax First”, this is a syntactic question, though its answer has consequences for truth.\textsuperscript{22}

\textsuperscript{22}For some influential related thoughts, see [N. D. Belnap, ’62].
For a logical constant $c$ of language $L$, let $c$’s introduction-package [elimination-package] in $L$ be the set of language-transcendent introduction-rules [elimination-rules] governing $c$ in $L$. We do need sets; an expression of Disjunction has a two-membered introduction-package, and an expression of Biconditionality has a two-membered elimination package; an expression of Surd has the empty introduction package. Let $c$’s package-pair in $L$ be the ordered pair of its introduction-package and its elimination-package. Properly speaking, this is the logical concept that $c$ expresses in $L$. Supplementing Thesis 3, I propose

Thesis 3': If $c$ is a logical constant in $L$, either all of $c$’s introduction rules are among those that overtly constitute $c$’s sense, or all of $c$’s elimination rules are.

Our question now is: what package-pairs are logical concepts? Most obviously, $c$’s elimination package must invert its introduction package. This generalizes Prawitz’s “Inversion Principle”, an explication of one of Gentzen’s great ideas, the one behind both cut-elimination for sequent calculi and normalization for ND systems: if one reasons properly, one gain nothing by introducing a logical constant only to eliminate it. The rigorous idea is best expressed algebraically; I’ll forgo details here. Of course the elimination packages for the Big Five and for the standard quantifiers invert their corresponding introduction packages. Prior’s ‘tonk’ doesn’t express a logical concept because ‘tonk’-Elim doesn’t invert ‘tonk’-Intr.

Indeed, I think that we need perfect inversion: $c$’s elimination package is the maximum inverter of $c$’s introduction-package, and the latter is the maximum inverter of the former. (Tennant calls this “the requirement of harmony”.23) The ordering here is the natural ordering by strength on the appropriate sets of packages. Call a package-pair meeting these conditions “perfect”. The package-pairs for the Big Five and the universal and existential quantifiers are perfect.

Along with Thesis 3’, perfect inversion helps secure whatever constitutive rules tacitly govern $c$ on the basis of those overtly governing $c$. For if $c$’s introduction [elimination] rules are among its overtly sense-constituting rules, that introduction [elimination] package uniquely determine the rest of $c$’s sense-constituting rules: they are the members of its maximum inverter [inverteee]. Contrast the sophisticated constructivists with another tribe, the unsophisticated constructivists. (For this discussion, their constructivism doesn’t matter.) They use the “non-proviso” rules, Universal Elimination and Existential Introduction, without problems; for them, only these rules overtly govern ‘$\forall$’ and ‘$\exists$’. But they (like many students in introductory logic courses) haven’t really got the hang of Universal Introduction

23See [Neil Tennant, 2002], p. 316, 321.
or Existential Elimination, rules that involve those nasty provisos. In other words, the latter rules do not overtly govern ‘∀’ and ‘∃’ in their language. Maybe they even had some difficulties with Disjunction Elimination (like some students in introductory logic classes), or Conditional Introduction. Some of their great mathematicians managed to use the problematic quantifier-rules correctly in proofs that others of the tribe could come to find persuasive, but without having achieved any explicit formulation of these rules. This tribe is rather like the Europeans of the late 18th century; in fact, one of their famous philosophers attributed the apparent cogency of these proofs to “pure intuitions”, experiences that this philosopher said were essential parts of understanding these proofs.

Our radical translator might have a harder time with these unsophisticated constructivists than with their more sophisticated cousins. But if our translator also a logician, she or he has reason to think that in their language Universal Introduction and Existential Elimination tacitly govern ‘∀’ and ‘∃’: the former is the maximal inverter of Universal Elimination, and the latter is the maximal inverter of Existential Elimination. This tacit governance among actual logic students is shown by the fact that many such students at first find Universal Introduction and Existential Elimination puzzling and ad-hoc, but with proper teaching they come to find them natural, even primitively compelling, and don’t think that they have been taught new meanings for old words. (Consider these two facts: that Universal Introduction and Universal Elimination form a perfect pair, and that Universal Elimination overtly primitively governs the unsophisticated constructivist’s use of ‘∀’. Do these facts suffice to make Universal Introduction tacitly govern ‘∀’ among the unsophisticated constructivists? I won’t rule this out, though my characterization of what it is for a rule to tacitly govern a constant contained the clause concerning the disposition to learn in order to avoid ruling it in.)

Still, perfection isn’t enough. Call a package-pair ⟨I, E⟩ definitive iff for any two constants c and c’ in any language L, if L’s lexicon assigns both to the package-pair ⟨I, E⟩, then c and c’ are provably equivalent using only rules in I ∪ E. E.g. if the package-pair is designed for n-place formula connectives, equivalence means that for any formulas ψ₀, ..., ψ_{n-1} of such a language, □c(ψ) ⇒ c’(ψ) and □c’(ψ) ⇒ c(ψ) are provable. With the notion of definitiveness on the table, I’ll stick my neck far out and suggest:

Thesis 6. Perfection and definitiveness are necessary and sufficient for a package-pair to be a logical concept.
§10. So far I’ve considered logical constants with regard to their sense. But a theory of sense needs what Peacocke calls a determination theory to characterize how the sense of an expression, or better the conditions for grasp that sense, contribute to determining the expression’s “referent” or, perhaps less misleadingly, its semantic value. Peacocke coined the phrase ‘determination theory’ with regard to concepts, not linguistic expressions; but language as well as thought needs a determination theory, even if language somehow inherits its determination theory from thought.

I’ll assume that a semantic theory, whatever else it does, assigns linguistic expressions to semantic values, and that this assignment capture how that expression contributes to determining at least the truth- and falsity-conditions of statements in which it occurs. (In a loose sense, this Fregean picture of semantic theory is “Realistic”. But it carries no commitment to thinking that concepts of truth and falsity are the central concepts of any plausible semantic theory, or to the thesis that understanding every truth-apt statement consists in “knowing its truth-conditions.” It is not obvious that this Fregean framework applies to a language whose total logic is constructive; here I merely proceed on the hypothesis that it does.) Much is unclear about what semantic values should be, especially for a language whose total logic is constructive. It’s conceivable that an unambiguous logical constant has distinct constructive and classical semantic values; perhaps this is the kernel of truth behind the popular view that Thesis 5(2) contests. Note that this would not compromise Thesis 5(2), since there is no road from reference back to sense. (If the best determination theories for constructive and for classical discourse have this result, it seems likely that the classical semantic values will be “restrictions” or special cases of the constructive semantic values.)

Think of a truth- or falsity-condition as a function, perhaps partial, from possible situations (or worlds “of evaluation”) to truth-values. To handle statements containing variable-binding constructions, we need to look beyond truth and falsity to satisfaction and frustration. So given a variable-assignment, let’s say that there are (at least) two satisfaction-values; given a variable-assignment and a possible situation or world, the semantic value of a formula will determine a satisfaction-value for that formula relative to these given. We demand at least this of the semantic value of an expression: it must capture how that expression contributes to the satisfaction- and frustration-conditions (hereafter the pre-alethic conditions) for formulas in which that expression occurs.

Let’s set aside the deep question of how best to conceive of semantic values, and consider what might be a narrower question: how do the rules constituting
the sense of a logical constant help determine its contribution to the pre-alethic conditions for formulas in which it is the main logical constant? Call this aspect of its semantic value its contributory value. By themselves, a logical constant’s sense-constituting rules doesn’t determine its contributory value. They do so only together with certain constraints on satisfaction and frustration.

Here are some appealing constraints for any language that people might use for communication or thought:

no formula is both satisfied and frustrated;
some formula is frustrated;
for two formulas expressing the same sense, one is satisfied iff the other is;
ditto for frustration.

Suppose that we’ve settled on enough of our determination theory regarding speakers of $L$ to specify the pre-alethic conditions for the “logic-free” formulas of $L$, those containing no logical constants; and suppose that this specification honors the above constraints. We now want to extend that theory to the remaining formulas of $L$.

Let a substitution instance of a formula-inference $\Delta \Rightarrow \varphi$ be a formula-inference obtainable from $\Delta \Rightarrow \varphi$ by uniform substitution of expressions of appropriate type for non-logical constants, and by restrictions of bound variables for variable-binding operators. Let $\Delta \Rightarrow \varphi$ be sound [cosound] iff each of its substitution instances preserves satisfaction [non-frustration], i.e. if all of members of its premises (i.e. antecedent) are satisfied [non-frustrated] then so is its conclusion (i.e. succedent). (This follows the mathematical usage of ‘sound’. Many philosophers use ‘valid’ to mean what I’ll mean by ‘sound’; but others mean something different, e.g. counterfactual preservation of warranted assertibility, or of knowledge, or something else. At least mathematical usage has been pretty unambiguous.) Let a sequent inference be sound [cosound] iff it preserves soundness [cosoundness], i.e. if its premises are all sound [cosound] then so is its conclusion. These semantic properties apply as well to entire arguments, in the obvious way. A rule is sound [cosound] in $L$ iff all inferences in $L$ that instantiate that rule are sound [cosound].

Let Basic Soundness [Cosoundness] be this requirement on satisfaction and frustration in $L$:

every argument constructed using only sense-constituting deductive rules that governing logical constants of $L$ is sound [cosound].
Let Total Soundness [Cosoundness] be the corresponding requirement for arguments constructed using any deductive rules that govern logical constants in \( L \). Basic Soundness and Cosoundness strike me as compelling constraints on how a determination theory assigns pre-alethic conditions to formulas of \( L \). I’m less confident that we must insist on Total Soundness and Cosoundness. Of course if \( L \)’s logic is classical and \( L \) is sufficiently expressive, anti-realists of a Dummettian stripe will say that satisfaction in \( L \) just won’t satisfy Total Soundness: \( L \)’s speakers are in a state of philosophical error; \( L \) isn’t bivalent, and \( EM_L \) just ain’t sound. (Note: if \( L \) contains logical constants expressing Negation and Disjunction and \( L \)’s total logic is classical, Total Soundness implies that \( L \) is bivalent: for any statement \( \varphi \) in \( L \), Classicality insures that there is a proof of \( \neg \varphi \vee \neg \varphi \); Soundness requires that \( \neg \varphi \vee \neg \varphi \) be true-in-\( L \); constructive reasoning gets us that either \( \varphi \) is true-in-\( L \) or \( \neg \varphi \) is true-in-\( L \), which implies that \( \varphi \) is either true-in-\( L \) or false-in-\( L \).) Still, one might conjecture this: if satisfaction and frustration honor Total Soundness and Cosoundness as well as the obvious constraints, then the sense-constituting rules for any logical constant will suffice to uniquely fix that constant’s contributory value. For a stronger conjecture, replace ‘Total’ by ‘Basic’.

As long as we restrict sense-constituting rules to the familiar introduction and elimination rules for \( L \)’s logical constants, these conjectures are false. Consider Conjunction: without assuming bivalence, \( \&\text{-Intr}_L \) and \( \&\text{-Elim}_L \) are sound both for Weak Kleene (aka Fregean) and Strong Kleene Conjunction; so these rules don’t uniquely determine the contributory value of ‘\&’. To avoid this trivialization, without considering “bi-modal” rules, we could weaken the above conjectures by adding the constraint that \( L \) be bivalent:

\[
\text{any formula is either satisfied or frustrated.}
\]

I will argue that, even so weakened, these conjectures are false.

Let’s look at the simplest sort of logical constant: an \( n \)-place extensional connective \( c \). Here extensionality is a proof-theoretic property. First, for any set \( \Delta \) of formulas, let any formulas \( \varphi \) and \( \varphi' \) be equivalent mod \( \Delta \) iff \( \varphi' \) is derivable from \( \Delta \cup \{ \varphi \} \) and \( \varphi \) is derivable from \( \Delta \cup \{ \varphi' \} \), this using only sense-constitutive rules. Let \( c \) be extensional iff for any such \( \Delta \) and any formulas \( \varphi_0, ..., \varphi_{n-1} \) and \( \varphi'_0, ..., \varphi'_{n-1} \), if \( \varphi_i \) and \( \varphi'_i \) are equivalent mod \( \Delta \) for each \( i \in n \), so are \( c(\varphi_0, ..., \varphi_{n-1}) \) and \( c(\varphi'_0, ..., \varphi'_{n-1}) \).

Suppose \( c \) is an extensional logical constant in \( L \). Rather than require specification of \( c \)'s contributory value, let’s merely ask that an acceptable determination theory imply that \( c \) is weakly truth-functional (“satisfaction-functional” is more
accurate, but I’ll stick with the more familiar phrase): for any variable-assignment, any possible situation, and any formulas \( \varphi_0, \ldots, \varphi_{n-1} \) in \( L \),

if each of \( \varphi_0, \ldots, \varphi_{n-1} \) has a satisfaction-value then these values uniquely determine \( \Gamma c(\varphi_0, \ldots, \varphi_{n-1}) \)'s satisfaction-value.

(Note: weak truth-functionality contrasts with strongly truth-functionality, which in addition to weak truth-functionality requires this: if \( \Gamma c(\varphi_0, \ldots, \varphi_{n-1}) \) has a satisfaction-value then \( \varphi_0, \ldots, \varphi_{n-1} \) have one of the distributions of satisfaction-values that determine \( \Gamma c(\varphi_0, \ldots, \varphi_{n-1}) \) to have that satisfaction-value. Bivalence with weak truth-functionality insures strong-truth-functionality; but without bivalence, it doesn’t. In Intuitionistic Logic each of the standard connectives is extensional and weakly truth-functional, but Conditionality isn’t strongly truth-functional. E.g. \( (\varphi \supset \varphi) \) is true though \( \varphi \) may be neither true nor false. Also, without bivalence the Weak Kleene connectives are strongly truth-functional, but Strong Kleene Conjunction and Disjunction are not.) We can now formulate a well-defined test: if the sense-constituting rules for an extensional connective \( c \), together with our general constraints on satisfaction and frustration determine \( c \)'s contributory value, then they must imply that \( c \) is weakly truth-functional. Focussing on familiar extensional connectives, do they do that?

§11. To make the issues vivid, let’s return to our the radical translator. She or he has determined the pre-alethic conditions for logic-free formulas of \( L \), and now wants to determine them for the rest. For generality, we won’t allow her to assume bivalence for \( L \).

For Conjunction matters are straightforward: regardless of what other logical constants \( L \) contains, Soundness and Cosoundness insure that ‘&’ is weakly truth-functional. Other connectives are more problematic. It’s useful to consider Negation; from its weak truth-functionality we can show the weak truth-functionality of other familiar connectives expressible in \( L \).

Suppose we’re given a variable-assignment. By our second constraint, it frustrates some formula; suppose it’s \( \theta \). The cosoundness of \( \bot \Rightarrow \theta \) implies that \( \bot \) is frustrated. Then the cosoundness of \( \varphi, \Gamma \neg \varphi \vdash \bot \) requires that either \( \varphi \) or \( \Gamma \neg \varphi \) be frustrated. Our first-mentioned constraint gives these principles: if \( \varphi \) is satisfied then \( \Gamma \neg \varphi \) is frustrated; if \( \Gamma \neg \varphi \) is satisfied then \( \varphi \) is frustrated.

But if our determination theory is to declare ‘\( \neg \)’ to be weakly truth-functional, it had better provide this crucial principle: if \( \varphi \) is frustrated then \( \neg \varphi \) is satisfied. Peacocke recognizes that this involves a step “beyond the primitively obvious”,
that this “raises the question of how the thinker knows such principles”, and that “the issues deserves extended attention.”

(He considers a thinker reflecting on her own concepts, not the radical translator; still, the issue is the same.) (One might prefer to consider connectives simpler than Negation, i.e. of level 0 in the hierarchy. The corresponding non-obvious principles for ‘∨’ and ‘⊃’ are: if \( \varphi \) and \( \psi \) are frustrated then so is \( (\varphi \lor \psi) \); if \( \varphi \) is frustrated then \( (\varphi \supset \psi) \) is satisfied.)

Here’s the crucial point: Soundness and Cosoundness, with our other above-mentioned constraints, do not insure this non-obvious principle; adding Bivalence doesn’t help. So the sense-constituting rules for ‘\( \neg \)’ together with these constraints don’t imply that ‘\( \neg \)’ is weakly truth-functional, let alone determine a unique contributory value for ‘\( \neg \)’.

A cheap proof. Let \( V \) be a truth-assignment (i.e. \( V \) maps the set of sentence constants into \( \{0, 1\} \), with 0 representing falsity and 1 representing truth) on the sentence-constants of sentential formal language respecting the standard truth-structures; suppose \( V(‘P’) = 0 \). We construct a “truth”-assignment \( V’ \) on the set \( Sent \) of sentences, one with respect to which all classical truth-functional derivations are sound but with \( V’(‘P’) = V’(‘\neg P’) = 0 \). Let \( V_0 \) be the usual extension of \( V \) to \( Sent \). From each minimal set \( \Delta \) classically implying ‘\( \neg P \)’ with \( V_0 \models \Delta \), select a \( \varphi \in \Delta \) and set \( V_1(\varphi) = 0 \). (Note that such \( \Delta \neq \{\} \).) For all other \( \varphi \in Sent \), set \( V_1(\varphi) = V_0(\varphi) \). From each minimal set \( \Delta \) classically implying some \( \psi \) so that \( V_1(\psi) = 0 \) but for all \( \varphi \in \Delta \) \( V_1(\varphi) = 1 \), select a \( \varphi \in \Delta \) and set \( V_2(\varphi) = 0 \), etc. Let \( V’ = \lim_{n \in \omega} V_n \). For any \( \Delta \) any \( \psi \), if \( \Delta \) classically implies \( \psi \) and for all \( \varphi \in \Delta \) \( V’(\varphi) = 1 \), then \( V’ \models \psi \), by the construction of \( V’ \). Since \( dom(V’) = Sent \), bivalence is satisfied. So Soundness and Cosoundness are satisfied.

The difficulty here would have been a rather good reason for Peacocke to retreat from deductive rules to rules of transition in his discussion of Negation in [Christoper Peacocke, ’86], one better than his given reason, viz. respect for Dummett’s requirement (mentioned in §4). But I’m unpersuaded we must retreat. What further constraints should we impose?

Here’s a bad idea. Note that in addition to its set of provable formula-inferences in \( L \), a logic realized in \( L \) determines a set of provable sequent inferences of \( L \),

\(^{24}\)In [Christoper Peacocke, ’87].

\(^{25}\)When I first noticed this, I thought it was a great discovery; a few weeks later I read [Christoper Peacocke, ’87]. Mark Brown told me that years ago he also thought he had discovered this point, but later found it discussed in notes he had taken years earlier for a class taught by Gerald Massey. I’d be grateful for any information about publications prior to Peacocke’s that make this point.
those of the form $\langle D, \Delta \Rightarrow \varphi \rangle$ for which $\Delta \Rightarrow \varphi$ is derivable from $D$. Let a variable assignment satisfy a formula-inference $\Gamma \Rightarrow \varphi$ iff it either frustrates some member of $\Gamma$ or satisfies $\varphi$, and let it frustrate $\Gamma \Rightarrow \varphi$ iff it both satisfies all members of $\Gamma$ and frustrates $\varphi$. Let Super-soundness [Super-cosoundness] be the constraint that provable sequent inferences preserve satisfaction [non-frustration]. If our variable assignment frustrates $\varphi$, it satisfies the inference $\{ \varphi \} \Rightarrow \bot$, from which $\Rightarrow \neg \neg \varphi$ is derivable. Assuming Super-soundness, $\Rightarrow \neg \neg \varphi$ is satisfied; since no member of $\{ \}$ is frustrated, $\neg \neg \varphi$ is satisfied.

This approach treats formula-inferences as if they were formulas; it replicates Russell’s unfortunate confusion of conditionality and implication properly-so-called, viz. entailment. An inference isn’t true; so calling one satisfied or frustrated is ill-motivated. In fact we don’t want satisfaction to conform to Super-soundness! ‘$\forall$’ Introduction permits us to infer $\Rightarrow \forall x P x$ from $\Rightarrow P x$ (since ‘$x$’ doesn’t occur free in any member of $\{ \}$ or in ‘$\forall x P x$’). But we don’t want satisfaction of the latter to require satisfaction of the former.

Now for a better idea. Let’s first consider English. How can we justify this proposition: if ‘That dog is sleeping’ (accompanied by demonstration of my dog) is false-in-English then ‘That dog isn’t sleeping’ is true-in-English? Assume the if-clause. Using ‘false-in-English’ elimination, we can conclude that my dog isn’t sleeping. By ‘true-in-English’ introduction, we can conclude to the then-clause. Applying conditional-introduction, we’re done. This is argument by semantic descent followed by ascent.

Now suppose English is our radical translator’s home language. Suppose that $\varphi$ is a statement of $L$, so truth and falsity may replace satisfaction and frustration; suppose it’s atomic. Our translator understands $\varphi$, setting $\text{trans}_\varphi =$ the statement made in her current context by ‘That dog is sleeping’ accompanied by a pointing gesture towards a dog. Our translator may reason as follows. “Assume that $\varphi$ is false-in-$L$. Thus, using ‘false-in-$L$’ introduction, that dog isn’t asleep. ‘That dog isn’t asleep’ is a negation of ‘That dog is asleep’. I’ve determined that ‘$\neg$’ is governed in $L$ by the same package-pair that governs expressions of Negation in my English. A philosopher has persuaded me that this insures that they express the same sense; so I can translate ‘$\neg$’ as an expression of Negation. So $\text{trans}_\neg \varphi$ is the statement that’s expressed in my current context by ‘That dog isn’t asleep’. Since that dog isn’t asleep, $\neg \neg \varphi$ is true-in-$L$, by ‘true-in-$L$’-introduction.”

This pattern of argument generalizes to any atomic formula that our translator understands. Suppose for the moment that the only logical constants in $L$ are extensional connectives. Such arguments then will give our translator the pre-
alethic conditions for formulas of $L$ of logical depth 1. Iterating by depth will secure the desired principle for any formula of $L$.

Note this. I suggested that our translator can in principle figure out the sense of a logical constant of $L$ without understanding a single formula of $L$. In contrast, the above justification for the non-obvious principle requires our translator to understand the formulas of $L$ to which an extensional connective applies well enough to translate them, starting with the atomic formulas and bootstrapping up. But semantic descent and ascent would be available no matter how $\varphi$ was translated – even if mis-translated! Our translator’s reliance on such arguments doesn’t force us to say that the semantic value of ‘$\neg$’ depends on the senses of the atomic formulas of $L$ or on their semantic values.

Still, the above argument may produce suspicion. Is this argument non-explanatory? Does it drives the problem posed by the non-obvious principle from $L$ to English, or even to Thought? If it assumed that an expression of Negation in English reversed truth-value, this charge would stick. But it uses no claims about the truth- or falsity-conditions of English sentences or of the translator’s thoughts. Its dialectical status is delicate. One might think this: if one is unsure whether the fact that ‘$\neg$’ in $L$ is governed by the rules governing expressions of negation in English give one a good reason to translate ‘$\neg$’ as an expression of Negation, then we’d look to our determination theory to settle whether we should so translate ‘$\neg$’. But if our determination theory is supported in part by arguments like the above, that would be illegitimate. That seems right; but I don’t think that we should be unsure in the way the if-clause suggests; so I don’t think that we need a determination theory to justify the translation of ‘$\neg$’. It is legitimate to use one’s theoretical beliefs about senses to support one’s preferred determination theory. The latter theory for a language $L$ must be tailored to an account of sense-grasping for expressions of $L$; so what could be wrong with relying on the latter account in one’s justifications for principles of a determination theory? “Syntax first” is a part of an account of sense-grasping; and it supports our linguist’s translation of ‘$\neg$’ as an expression of Negation”. I submit that the fact that “Syntax first” helps justify the non-obvious principle, and others like it, constitute abductive support for “Syntax first”.

Enough for connectives; what about variable-binding logical constants? The notion of extensionality generalizes straightforwardly to them. And we may extend the notion of weak truth-functionality to such constants; I’ll refrain from details. Suffice to say: even with the weak truth-functionality of the familiar connectives in place, our difficulty with Negation has analogues for expressions
of first-order Universal or Existential quantification. For example, this principle is non-obvious: if every \( \nu \)-variant of a variable-assignment satisfies \( \psi \) then that assignment satisfies \( \forall \nu \psi \). It can be proved that even assuming bivalence, Soundness and the other obvious constraints can’t deliver this non-obvious principle. (Peacocke claims that universal quantification over the natural numbers is “the unique second level concept” whose possession condition is this: finding Universal Elimination using substitution with appropriate numerical concepts primitively compelling.\(^{26}\) This passage seems to confuse the concept of Universality over the natural numbers with its semantic value, the universal quantifier restricted to the natural numbers. It immediately follows a discussion of Conjunction; I’m not sure whether Peacocke meant to suggest that a determination theory’s account of the concept of Universality over the natural numbers would be as straightforward as its account of the concept of Conjunction. Suffice to say that it isn’t.) An argument by semantic descent and ascent can secure this principle; I’ll spare the reader the details.

Vague Conjecture 2: Once we have settled on what sort of semantic values logically atomic expressions have, Total Soundness and Cosoundness, with the obvious constraints on satisfaction and frustration and (here’s the crucial point) all instances of the Sat and Fr (and thus the Tr and Fa) schemata, will suffice to determine the semantic values, or at least the contributory values, for \( L \)’s logical constants.

Should we strengthen this conjecture by replacing ‘Total’ by ‘Basic’? Such a strengthening would imply that logical constants have the same semantic values for both classical and constructive discourse. (More fully: if \( L \) and \( L' \) differ merely in that the total logic for one is classical and for the other is constructive, then each logical constant in either (i.e. in both) has the same semantic value in \( L \) and in \( L' \).) I have no settled opinion regarding this strengthened conjecture, though I am inclined to reject it.\(^{27}\)

\(^{26}\)In [Christopher Peacocke, ’92], p. 7.

\(^{27}\)Thanks to audiences at Notre Dame, Syracuse University, St. Andrews and in my own department, and particularly Michael Detlefsen, Michael Fara, Michael Kremer, Jeff Roland, and Zoltan Szabo, for valuable comments and discussions.
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